# மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள் நடத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 7வது முன்னோடிப் பரீட்சை - 2016

## கல்விப் பொதுத் தராதரப் பத்திர(உயர் தர) முன்னோடிப் பரீட்சை - 2016
## General Certificate of Education (Adv.Level) Pilot Examination - 2016

| இணைந்த கணிதம் II<br>Combined Maths II | 10 | E | II | மூன்று மணித்தியாலம்<br>*Three hours* |
|---|---|---|---|---|

| *Index Number* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|

**Instructions** :

* *This question paper comprises Part* **A** *(Questions 1 – 10), Part* **B** *(Questions 11 – 17)*
* **Part A:**
  *Answer all questions on this paper itself. Write your answers in the space provided for each questions. If the space provided is insufficient, extra sheets can be used.*
* **Part B:**
  *Answer* ***five*** *questions only. Use the papers supplied for this purpose.*
* *At the end of the time allotted for this paper, tie the two papers together so that Part* ***A*** *is on the top of Part* **B** *before handing over to the supervisor.*
* *You are permitted to remove only Part* **B** *of the question paper from the examination hall.*
* *In this question paper* ***g*** *denotes gravitational acceleration.*

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| (10) Combined Maths I | | |
|---|---|---|
| **Part** | **Question No.** | **Marks** |
| **A** | 1 | |
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| Paper I | |
| Paper II | |
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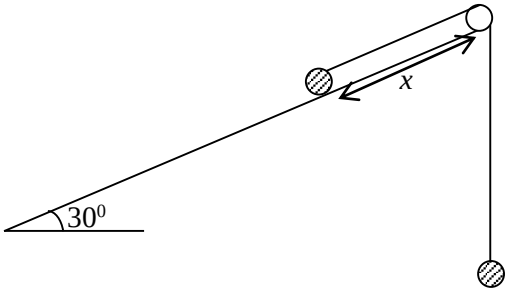
[ *see page two*

## Part– A

**01.** Two particles *A* and *B* of mass *4m*, *m* respectively are connected by a light inelastic string of length *l* passing over a light smooth pulley C fixed at the vertex of a smooth inclined plane inclined $30^0$ with horizontal. The system is released from rest. When *A* is kept on the inclined plane near the pulley and *B* hangs freely. When the particle A has moved a distance *x* along the inclined plane, show that the speed of each particle becomes *v* where $v^2 = \frac{2gx}{5}$ using the principle of conservation of energy. **Hence** or otherwise find the magnitude of acceleration of the system.

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**02.** A fire cracker blasts and starts to move in a direction inclined $30^0$ with horizontal with a uniform acceleration of $\frac{9}{2} ms^{-2}$ from rest from a point on a horizontal ground. It moves with this acceleration for $\frac{20}{3} s$, then move under gravity. Here $g = 10 ms^{-2}$ Find the speed of the cracker at $\frac{20}{3} s$, and show that the cracker reaches the ground after 5 seconds.

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**03.** A vehicle of mass $M\ kg$ can reach its maximum velocity of $ums^{-1}$ on a straight smooth road. When the vehicle moves upwards along an inclined road of $\alpha$ inclination, it can reach the maximum velocity of $vms^{-1}$. The resistive force R is always $R = kw^2 N$. Here $w$ is the speed of the vehicle in $ms^{-1}$ and $k$ is a constant. Show that $k = \frac{mgv\sin\alpha}{u^3 - v^3}$

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**04.** Two spheres $A, B$ of same mass and same radii moves with the speed of $3ms^{-1}$ and $5ms^{-1}$ respectively opposite to each other and collides directly. If the direction of B is reversed after the collision, show that $e > \frac{1}{4}$. Here the co- efficient of restitution between the spheres is $e$.

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**05.** The position vectors of points $A$ and $B$ are $\underline{a} = \underline{i} + 2\underline{j}$, $\underline{b} = 2\underline{i} - \underline{j}$ respectively, about the origin O. The points C and D are selected as $\overrightarrow{OC} = \lambda\underline{a}$ and $\overrightarrow{OD} = \mu\underline{b}$ when $\lambda > \mu$. By finding the vectors $\overrightarrow{AB}, \overrightarrow{CD}$, when $AB \perp CD$ and the magnitude of $\overrightarrow{CD}$ is $2\sqrt{10}$, find the values of $\lambda$ & $\mu$.

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**06.** A smooth hemisphere of radius $a$ is fixed as its circular base is kept on the ground. A uniform rod $AB$ of length $2a$ is beared as its one end $A$ is on ground and a point $C$ on the rod is in contact with the curved surface of the hemisphere and makes angle $\theta$ with horizontal and the rod is in equilibrium. The rod and the center of the hemisphere is in a same vertical plane. Show that the co- efficient of friction $\mu$ between the rod and the ground satisfies the inequality $\mu \geq \frac{9}{13}$. Here $Sin\theta = \frac{3}{5}$.

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**07.** *A, B, C* are three exhaustive events in a limited sample space. *A* and *B* are mutually exclusive and *A* and *C* are mutually exclusive. If $P(A) = 6x$, $P(B) = P(C) = x$, $P(B \cap C) = y$. Using the relation $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) +$ . $P(A \cap B \cap C)$ Find *y* in terms of *x*. Deduce that $\frac{1}{8} \leq x \leq \frac{1}{7}$.

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**08.** let *A*, *B* and *C* are three events in a sample space *S*. In the usual notations, if $P(P/C) \geq P(B/C)$ and $P(A/C^1) \geq P(B/C^1)$, show that $P(A) \geq P(B)$.

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**09.** The frequency distribution of 150 data is shown below.

| ($x$) | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|
| Frequency ($f$) | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ |

The relationship between ($x$) and frequency (f) is $f = kx$. Here $k$ is a constant. Find the mean value of this distribution.

**10.** A set S of 10 whole numbers is arranged in ascending order as given. $S = \{1,3,8,8,a,b,c,d,e,f\}$. The mean value of these numbers is 8 and there is only one mode that is 9. The last three numbers are consecutive whole numbers. Find the values of $a,b,c,d,e \& f$.

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கல்விப் பொதுத் தராதரப் பத்திர(உயர் தர) முன்னோடிப் பரீட்சை - 2016
General Certificate of Education (Adv.Level) Pilot Examination - 2016

| இணைந்த கணிதம் II<br>Combined Maths II | 10 | E | II |
|---|---|---|---|

**Part –B**

- Answer **five** questions only.

**11.** (a) There is a horizontal ceiling at the height of $\frac{3h}{2}$ from a smooth horizontal ground. A Particle *P* is thrown vertically upwards with the speed *u* form a point at a height of $\frac{h}{2}$ from the ground. At the same moment, from the same height another particle *Q* is thrown vertically downwards with the speed *2u* under the gravity. Particle *P* just reaches the ceiling when $t = T_1$. Particle *Q* hits the ground having co-efficient of restitution of $\frac{1}{\sqrt{3}}$ and in the return motion it reaches the ceiling when $t = T_2$.

i. Show that $u^2 = 2gh$ and $T_1 = \frac{u}{g}$

ii. Show that the bouncing speed of *Q* just after hitting the ground is $\sqrt{\frac{3}{2}}u$

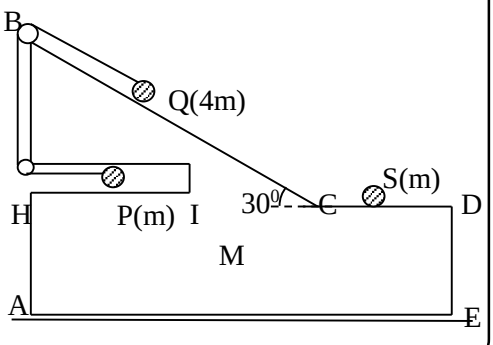
iii. Show that particle *Q* just touches the ceiling and also show that $T_2 = \frac{u}{\sqrt{2}g}\left(3 - 2\sqrt{2} + \sqrt{3}\right)$

iv. Draw the velocity – time curves of *P* and *Q* on the same diagram until $t = T_2$

v. Using the velocity – time curves, show that when *Q* reaches the ceiling, the particle *P* is at the depth of $\frac{3u^2}{4g}\left(\sqrt{3}+1-\sqrt{6}\right)^2$ down the ceiling.

(b) For a cyclist moving towards South with the constant velocity *u*, the wind seems to flow in a direction *West θ North*. When the same speed, he observed that the wind is flowing in a direction *West β North*. Show that, for the cyclist when he rides towards North with the speed *2u*, the wind would be seemed to flow in a direction *West α North*. Here $2\tan\alpha = 3\tan\beta - \tan\theta$. Determine the speed of wind.

**12.**(a) The figure *ABCDE* shows the vertical cross –section view of a block of mass *M*. A smooth cavity *HI* is hollowed inside the block parallel to *AE*. *AB, BC, CD* are the maximum inclined lines along respective faces. *BC* makes $30^0$ with horizontal. The block is kept on a smooth fixed horizontal table where the face *AE* touches the table. Two particles *P* and *Q* of masses *m* and *4m*

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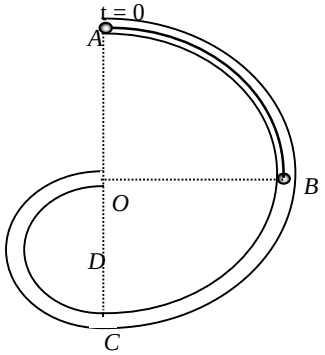
respectively are kept on *HI* and *BC* respectively and connected at the ends of a light inextensible string which passes over two small smooth pulleys fixed at *H* and *B*. A particle *S* of mass *m* is kept on *CD*. The system is released from rest when the string is kept taut. Let the magnitude of acceleration of *P* and *Q* relative to the block is *f* and the acceleration of *S* along *CD* is $f^1$ relative to block. If the acceleration of the block relative to earth is *F* along *EA*. Write the equations of motion.

**Hence**, show that $F = \dfrac{2(2\sqrt{3}-1)mg}{5M + 4\sqrt{3}\left(\sqrt{3}+1\right)m}$

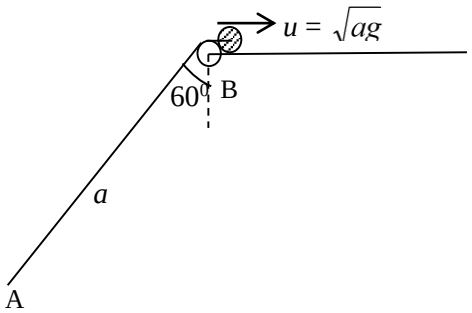
(b) A smooth narrow tube *ABC* is bent as shown in the figure. One part of the tube is a semicircle of radius of *a* of centre *O* and the other part is a semicircle with centre *D* of radius $a/2$. The tube is fixed in a vertical plane where *AOC* is vertical and perpendicular to *OB*. The particle *P* of mass *m* and particle *Q* of mass 2*m* are kept inside the tube and connected by a light inextensible string of length $\dfrac{\pi a}{2}$. At the beginning *P* is at *A* while *Q* is at *B* and they are released from rest when the string is kept taut. When the radius that passes through particle *P* makes an angle $\theta(<45^0)$ with *OA*, using the principle of conservation of energy show that $\left(\dfrac{d\theta}{dt}\right)^2 = \dfrac{2g}{3a}(1-\cos\theta+2\sin\theta)$.




**Hence**, fine the tangential acceleration of particle *P*. Show that the tension in the string is $\dfrac{2mg}{3}(\cos\theta-\sin\theta)$.

When $\theta = 30^0$ the string breaks suddenly. Find the velocity of particle *Q* at that moment. In the following motion of *Q*, when the radius going through *Q* makes an angle β with *DO*, show that the particle *Q* reaches the momentary rest. Here $\beta = \cos^{-1}\left(\dfrac{4-\sqrt{3}}{3}\right)$.

**13.** One end of a light extensible string of natural length *a* and modulus of elasticity *2mg* is tied to fixed point *A*. The string passes over a smooth pulley *B* fixed at the edge of a rough horizontal table located above the level of *A*. A particle *P* of mass *m* is attached to the other end of the string. The distance *AB* = *a* and the angle between *BA* and the downward vertical is $\dfrac{\pi}{3}$. Initially the particle *P* is kept near the pulley *B* on the table and thrown with speed $u = \sqrt{ag}$ to move horizontally along the table. The co-efficient of friction between the particle *P* and the table is $\dfrac{1}{2}$. Let the extension in the string when *t* = *t* is *x*. Show that the equation of simple harmonic motion of the particle *P* is given by $\ddot{X} + \omega^2 X = 0$. Here

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$X = x + \frac{a}{4}$, $\omega^2 = \frac{2g}{a}$. Considering the solution of this equation in the form $\dot{X}^2 = \omega^2\left(a_0^2 - X^2\right)$, show that the amplitude $a_0$ of this Simple harmonic motion is $a_0 = \frac{3a}{4}$. Find *BD* where the point *D* is the maximum displacement of *P* from pulley *B*. Show that the time taken by the particle *P* to reach *D* from *B* is $\sqrt{\frac{a}{2g}}\cos^{-1}\left(\frac{1}{3}\right)$. Show that the next further motion of *P* is another simple harmonic motion and find its center of oscillation.

Show that the particle *P* comes to permanent rest at *B* in the following motion and show that the total time taken by the particle in the whole motion is $\sqrt{\frac{a}{2g}}\left[\pi + \cos^{-1}\left(\frac{1}{3}\right)\right]$.

**14.** (a) *O, A* and *B* are three non co-linear points. Here *O* is the origin, $\overrightarrow{OA} = \underline{a}$ $\overrightarrow{OB} = \underline{b}$. The points *C* and *D* are selected as $\overrightarrow{OC} = \lambda\underline{a}$ and $\overrightarrow{OD} = \mu\underline{b}$ where $0 < \lambda < 1 < \mu$. The lines *AB* and *CD* intersects at *E*. By considering $\overrightarrow{AE}$ and $\overrightarrow{AB}$, show that $\overrightarrow{OE} = (l-\alpha)\underline{a} + \alpha\underline{b}$ Here $0 < \alpha < 1$

**Hence**, prove that $(\mu - \lambda)\underline{e} = \lambda(\mu - 1)\underline{a} + (1-\lambda)\mu\underline{b}$. Here $\overrightarrow{OE} = \underline{e}$

If *E* is the midpoint of *AB*, Show that $\frac{1}{\lambda} + \frac{1}{\mu} = 2$.

(b) The midpoints of the sides $AB, BC, CD, DA$ of a square $ABCD$ are $P, Q, R, S$ respectively. Along the directions $PQ, QR, RS, SP, AC, BD$ the forces 10, 20, 30, 40, λ, μ Newton are acting respectively.

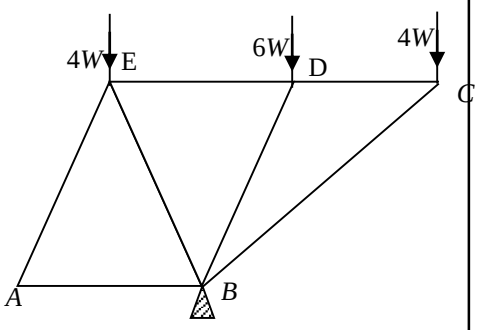
i. Show that the system cannot be in equilibrium for any values of λ and μ.

ii. If the system reduces to a couple, find the values of λ and μ

iii. If the system of forces is equivalent to a single force acting through *C*, Show that μ=70.

**15.** (a) Three uniform rods of *AB, BC* and *CD* each of length $2a$ and weight $W$ are jointed freely at *B* and *C*. The rod *BC* is horizontal and the end *A* and *D* are kept on a smooth horizontal plane. Two light, inextensible strings of same length are connected to the midpoints of rods *AB* and *CD* and the other ends are connected to the midpoint of rod *BC*. The strings are kept taut and *ABCD* is in equilibrium in a vertical plane. $\hat{ABC} = 120^0$. Show that the tensions in the strings is *2W*. Find the magnitude of the reaction at the joint *B* and show that the reaction makes an angle $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ with the horizontal

(b) The figure shows the frame work made by seven light rods jointed at their ends. The rods *AB, CD* and *DE* are horizontal. The all rods except *BC* are equal length. Three weights of $4W, 6W, 4W$ are hung from *C, D* and *E* respectively. The frame work is hinged at *A* and beared on a smooth peg at *B*. Show that the reaction at *B* is $21W$ and find the components of the reaction at *A*. Using the Bow's notation draw the stress diagram, find the stress on each rods and distinguish whether they're tension or thrust.

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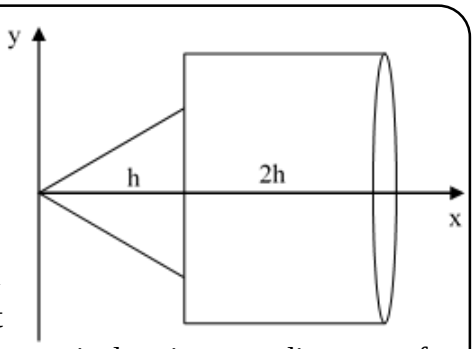
**16.** The density of a non – uniform solid cone of height $h$ is proportional to the distance perpendicular to axis from the line through the vertex. Show that the center of mass of this cone is at a distance $\frac{4h}{5}$ from the vertex $O$ along the axis.




A composite object is made using that cone and a uniform right cylinder of height $2h$ as shown in the figure. Show that the center of mass of this composite body is on the symmetrical axis at a distance of $\left[\frac{4m_1+10m_2}{5(m_1+m_2)}\right]h$ from the vertex of the cone. Here $m_1$ is the mass of the cone and $m_2$ is the mass of the cylinder.

(a) If $5m_2 \geq m_1$, show that the composite body would be in equilibrium when the curved surface of the cylinder touches a horizontal plane.

(b) If $5m_2 < m_1$, for the equilibrium of the composite body when the curved surface of cylinder touches a horizontal plane, find the minimum force that have to be applied on the vertex of the cone, upwardly perpendicular to the axis .

**17.** (a) A computer generates sums for children. The children should answer those questions with in the time limit. The probability of answering the first question correctly is **0.8**. When one question is answered correctly, the next question that generated will be harder than the previous. The probability of answering the new question correctly is **0.1** lesser than that of previous. When one question is answered incorrectly, there will be no change in the standard of the next question. The probability for answering the question correctly also will not be changed. When a student participates in this competition,

i. Find the probability of answering the second question correctly.

ii. Find the probability for that the second question had been answered correctly when it is given that the third question was answered correctly.

(b) The marks obtained by 100 students for a combined mathematics examination is given below.

| Marks | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|---|---|---|---|---|---|---|---|---|---|
| Number of students | 05 | 12 | 18 | 09 | 20 | 12 | 13 | 08 | 03 |

Find the mean, mode and the standard deviation of this marks distribution. Find the co-efficient of skewness and state the shape of this distribution.

The mean and the standard deviation of the marks obtained by above 100 students for a physics examination are 48 and 12 respectively. The marks ($x$) for the combined mathematics examination are made to a liner transformation as $y=ax+b$, in order to have the mean and standard deviation as equal to that of marks for physics examination here $a$ (>0) and $b$ are constants. By considering the values obtained for the mean and the standard deviation for the initial marks of combined mathematics, to their nearest whole numbers, find the values of $a$ and $b$. A particular student obtained 53 marks for combined mathematics and 50 marks for physics examination. Find, for which subject, is the student more intelligent.

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மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள்

நடாத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 7வது முன்னோடிப் பரீட்சை - 2016



# கல்விப் பொதுத் தராதரப் பத்திர(உயர் தர) முன்னோடிப் பரீட்சை - 2016
# General Certificate of Education (Adv.Level) Pilot Examination - 2016

| இணைந்த கணிதம் I<br>Combined Maths I | 10 | E | I | மூன்று மணித்தியாலம்<br>Three hours |
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| Index Number | | | | | | | | |
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- *At the end of the time allotted for this paper, tie the two papers together so that Part* ***A*** *is on the top of Part* **B** *before handing over to the supervisor.*
- *You are permitted to remove only Part* **B** *of the question paper from the examination hall.*

**For Examiner's Use Only**

| (10) Combined Maths I | | |
|---|---|---|
| **Part** | **Question No.** | **Marks** |
| **A** | 1 | |
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| **B** | 11 | |
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## Part-A

**1.** Using the method of mathematical induction, show that for all positive integers, when $2.7^n + 3.5^n$ is divided by 24, the reminder is 5.

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**2.** Let $f(x) \equiv x^2 - 4x + 3$. Draw the graph of $y = f(x)$. **Hence,** draw $y = |f(x)|$ on the same diagram. **Deduce** the range of $x$ where $f(x) < |f(x)|$.

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**3.** The point *P* denotes a complex number Z on the argon diagram. Draw the locus of *P* where $|Z+2+3i|=|1-Z|$ Find the minimum value of |z| in this locus and find Z at that time.

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**4.** In the binomial expansion of $\left(2x^3-\frac{3}{x^2}\right)^{15}$. Find;

a) The term that does not depend on *x*.
b) The sum of co-efficients in this expansion.

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**5.** Find the value of $\lim_{x \to 0} \frac{1-\cos^2(3Sinx)}{1-Cos2x}$.

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**6.** If the tangent drawn to the curve $y^2 = x(2-x)^2$ at the point (1, 1), intersects again that curve at the point *P*, find the co-ordinates of *P*.

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**7.** Find the equations of diagonals of parallelogram having $x-y+10=0$, $x+3y-2=0, x-y+2=0$ as its adjacent sides, without finding the co-ordinates of vertices.

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**8.** Find the centre and the radius of circle $S \equiv x^2+y^2-4x+6y-3=0$. If the circles $S=0$ and $S_1 = x^2+y^2+2x-2y+\lambda=0$ are touching each other, find λ.

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**9.** If the tangents drawn to the circle $S \equiv x^2 + y^2 + 2gx + 2fy + C = 0$ from a point $(x_0, y_0)$ are perpendicular to each other. Show that $g^2 + f^2 = x_0^2 + 2gx_0 + 2fy_0 + 2C$.

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**10.** If it is given that $Cos(\alpha + \beta) = 4/5, Sin(\alpha - \beta) = 5/13$ and $0 \le \alpha, \beta \le \pi/4$, by considering $2\alpha = (\alpha + \beta) + (\alpha - \beta)$, find tan 2α.

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கல்விப் பொதுத் தராதரப் பத்திர(உயர் தர) முன்னோடிப் பரீட்சை - 2016
General Certificate of Education (Adv.Level) Pilot Examination - 2016

| இணைந்த கணிதம் I<br>Combined Maths I | 10 | E | I |
|---|---|---|---|

**Part – B**

- Answer **five** questions only.

**11.** (*a*) *F(x), G(x)* and *H(x)* are the polynomials where *x* is in 4th grade

$F(x) \equiv (3x^2 - \alpha x + 3)(3x^2 - \beta x + 3)$. Here α, β are real constants

$G(x) \equiv x^4 - 3x^3 + 4x^2 - 3x + \lambda$

$H(x) \equiv x^4 + x^2 + 1$

(i) If $F(x) = 0$ and $G(x) = 0$ have same roots, find λ.

Show that the quadratic equation having roots α and β is $x^2 - 9x + 18 = 0$.

**Hence**, show that two of roots of $G(x) = 0$ are equal real roots and also show that the other two roots are imaginary.

(ii) If $F(x) \equiv 9H(x)$, find the possible values of α and β. Show that the roots of the equation $H(x)$ are **not** real .

(*b*) Let $f(x) \equiv ax^4 + bx^3 + cx^2 + x - 10$. Here $a,b,c \in \Re$. If *(x-1)* and *(x-2)* are factors of $f(x)$ and when $f(x)$ is divided by $(x+1)$, the remainder is 48, Find the values of *a,b,c*.

Find the remainder when $f(x)$ is divided by $(2x+1)$ and write $f(x)$ as the multiples of linear factors.

**12.** *(a)* A group of **5** students' representatives has to be selected initially from advanced level classes of a school. One of these 5 students should be the leader. These 5 students are selected from distinguished **5** science stream students (3 boys and 2 girls), **8** arts stream students (6 boys and 2 girls), **7** commerce stream students (4 boys and 3 girls) and **3** technology stream students (2 boys and 1 girls). The leader of the group should be a **science** stream student (a boy or a girl). The remaining 4 should be non-science stream students. Find the number of different ways the group can be created in the following instances.

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i. The group should has at least one arts stream boy and one arts stream girl.

ii. At least one student should be selected from each stream, but if two students are selected from same stream, one should be a boy and the other one should be a girl.

*(b)* Show that $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

Consider $\left[\frac{r(r+1)}{2}\right]^2 - \left[\frac{r(r-1)}{2}\right]^2$. Show that $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

For all positive integer r, in a series where r $^{th}$ term $U_r$ as a polynomial in terms of r, the sum of r $^{th}$ tem and $(r+1)^{th}$ term is $r(2r^2+3r-3)$

i. Show that $Ur = r^3 - 3r + 1$

ii. Find $\sum_{r=1}^{n} Ur$

**13.** *(a)* Three matrices *A, B, C* are given by $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ , $B = \begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$.

i. Show that $C^2 - 2C + 2I = O$ and find $C^{-1}$. Here I is an identity matrix and O is a 2x2 zero matrix.

ii. Find AB.

iii. Find the 2x2 matrix X satisfying the matrix equation CX –AB= O. Here O is 2x2 zero matrix.

*(b)* The complex number $Z_1 = 1, Z_2 = Cos\theta + iSin\theta$, here $0 < \theta < \pi/2$. On an Argon diagram, the points A and B denotes the complex numbers $Z_1$ and $Z_2$ respectively. Find the point *C* which denotes $(Z_1 + Z_2)$. **Hence**, Find the range of $|Z_1 + Z_2|$ and $Z_1 + Z_2$.

Write $(Z_1 + Z_2)$ in terms of $r(Cos\alpha + iSin\alpha)$. If $-\pi < \theta \leq \pi$, find the maximum of $|Z_1 + Z_2|$ and the corresponding $Z_2$. The complex number $Z = Z_1 + Z_2$ is depicted as $Z = x + iy$. Find the locus of Z when θ is changing.

**14.** *(a)* Using the first principles, prove that $\frac{d \tan x}{dx} = \sec^2 x$. Deduce that

$$\frac{d \tan^{-1} x}{dx} = \frac{1}{1+x^2}$$

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When $y$ is in terms of $x$, Show that $(1+x^2)\frac{dy}{dx} = \frac{dy}{d(\tan^{-1}x)}$

**Hence**, when $y = \frac{x}{\sqrt{1+x^2}} + Cos\left[2\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}\right]$

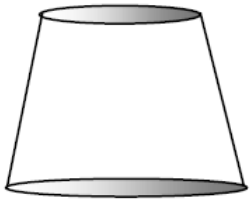
Using the substitution $\tan^{-1}x = \theta$ or otherwise show that $\frac{dy}{d(\tan^{-1}x)} = \sqrt{2}Cos\left[\frac{\pi}{4} + \tan^{-1}x\right]$

*(b)* Let $f(x) \equiv \frac{1+2x}{x(x+1)}$. Draw the rough graph of $y=f(x)$. By considering the intersection of the graph $y= f(x)$ and a straight-line $y = mx$, show that the third degree equation $mx^3 + mx^2 - 2x - 1 = 0$ has.

i. 3 real roots when $m>0$

ii. Only one real root when $m\leq 0$

(*c*) A straight tree trunk of circular cross section is 5m in length. This tree trunk is in the shape of a base of a cone. The radius of one base is 1.5 m and the other one's radius is 0.5 m. Show that the length of the cylindrical rod with the maximum volume that can be cut out from this tree trunk is 2.5 m.



**15.** *(a)* Using the partial fractions, find $\int \frac{x^2-3x+4}{(x-1)^2(x^2+1)}dx$

*(b)* Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Show that $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx = \int_0^{\pi} \frac{xSinx}{1+Sinx}dx = \pi\left(\pi/2 - 1\right)$

*(c)* When $A = \int_{-1}^{1} \frac{x^2}{1+e^x}dx$, and $B = \int_{-1}^{1} \frac{x^2}{1+e^x}$, using a suitable substitution show that $A=B$. Find the value of A+B. **Hence,** show that $A = B = \frac{1}{3}$

*(d)* Using the method of integration by parts, express $\int_0^1 x\ln(1+x/2)dx$ in the form of $a + b\ln\left(2/3\right)$ and find the values of $a$, $b$.

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**16.** Find the co-ordinates of mirror image of the point (α , β) which is on a straight line given by the equation $ax+by+c=0$.

The equation of two perpendicular bisectors of side *AB* and *AC* of a $\Delta ABC$ are $y+2x=0$ and $3y=x$ respectively. The centre of circumcircle of this triangle is *N*. *NA* passes through (2,2) and *BC* passes through the point (3,4)

*(a)* Find the co-ordinates of points *A,B* and *C*

*(b)* Find the equation of the circum circle of $\Delta ABC$

*(c)* Show that the equations of all circles passing through *A* and *N*, is in the form $x^2+y^2+2tx+(40-2t)y=0$ .Here *t* is a parameter.

**Hence**, find the equation of circle which having the minimum radius among those system of circles.

**17.** *(a)* Show that $\dfrac{Cos2x+Sin\left(\pi/4-x\right)}{Sin2x+\left(Cos\,\pi/4-x\right)+1}=\dfrac{Cosx-Sinx}{Cosx+Sinx}$

*(b)* State the sine rule for a triangle

In a quadrilateral $ABCD, B\hat{A}D=\pi/2, B\hat{A}C=\theta, A\hat{C}B=\alpha, AB=1, AD=m$ Apply sine rule separately to triangles *ABC* and *ADC* and show that $AC=\left(\dfrac{Sin(\alpha+\theta)}{Sin\alpha}\right)l=\left[\dfrac{Cos(\theta-\beta)}{Sin\beta}\right]m$.

**Deduce** that $Cot\theta=\dfrac{m-l\cot\alpha}{l-m\cot\beta}$

*(c)* Let $f(x)=4(Sin^4x+Cos^4x)$

Prove that $f(x)=3+Cos4x$

**Hence or otherwise**, draw the rough graph of $f(x)=4(Sin^4x+Cos^4x)$ for $|x|<\pi/2$, Calculate the area in between the curve $y=f(x)$ and the straight lines $x=\pm\pi/2$ and $y=2$.

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